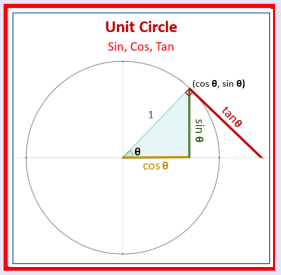
Math 241

Winter 2024

Lecture 13



Feb 19-8:47 AM

Class QZ 10

Find the measure of both angles if

$\tan\frac{1}{2}x = \cot\frac{1}{3}x$

Cofunctions are equal when angles are Complementary.

$\frac{1}{2}x + \frac{1}{3}x = 90$

LCD = 6

$3x + 2x = 6 \cdot 90$

$5x = 6 \cdot 90$ $\quad x = 108$

$x = \frac{6 \cdot 90}{5}$

$\frac{1}{2}(108) = 54°$

$\frac{1}{3}(108) = 36°$

Jan 23-11:57 AM

Verify

$\tan(x-45^\circ) + \tan(x+45^\circ) = 2\tan 2x$ ✓

Recall $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B}$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A \tan B}$

$LHS = \dfrac{\tan x - \tan 45^\circ}{1+\tan x \tan 45^\circ} + \dfrac{\tan x + \tan 45^\circ}{1-\tan x \tan 45^\circ}$ ($\tan 45^\circ = 1$)

$= \dfrac{\tan x - 1}{1+\tan x} + \dfrac{\tan x + 1}{1-\tan x}$

$= \dfrac{(\tan x - 1)(1-\tan x) + (\tan x + 1)(1+\tan x)}{(1+\tan x)(1-\tan x)}$

$= \dfrac{\tan x - \tan^2 x - 1 + \tan x + \tan x + \tan^2 x + 1 + \tan x}{(1+\tan x)(1-\tan x)}$

$= \dfrac{4\tan x}{1-\tan^2 x} = \dfrac{2 \cdot 2\tan x}{1-\tan^2 x}$ ✓

$= 2 \cdot \dfrac{\tan x + \tan x}{1 - \tan x \cdot \tan x}$

$= 2 \cdot \tan(x+x)$

$= \boxed{2\tan 2x}$ ✓

Jan 24-8:05 AM

write $2\cos 85^\circ \sin 140^\circ$ as sum or difference of two functions.

$2\cos 85^\circ \sin 140^\circ = 2 \cdot \frac{1}{2}\left[\sin(85^\circ+140^\circ) - \sin(85^\circ-140^\circ)\right]$

$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$= \sin(225^\circ) - \sin(-55^\circ)$

Recall $\sin(-\alpha) = -\sin\alpha$

$= \sin 225^\circ - -\sin 55^\circ$

$= \boxed{\sin 225^\circ + \sin 55^\circ}$

$225^\circ = 180^\circ + 45^\circ$

$\sin 225^\circ = \sin(180^\circ + 45^\circ)$

$= \sin 180^\circ \cos 45^\circ + \cos 180^\circ \sin 45^\circ$ ($\sin 180^\circ = 0$, $\cos 180^\circ = -1$)

$= -\frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{2}}{2} + \sin 55^\circ$

Jan 24-8:16 AM

Write $\cos 4x + \cos 8x$ as the product of two trig. functions.

$$\cos 4x + \cos 8x = 2\cos\frac{4x+8x}{2}\cos\frac{4x-8x}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$$

$$= 2\cos 6x\cos(-2x)$$

Recall

$\cos(-\alpha) = \cos\alpha$

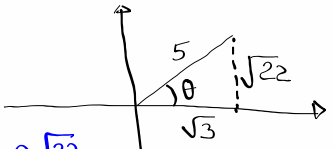
$$= \boxed{2\cos 6x\cos 2x}$$

Jan 24-8:23 AM

Find $\tan 2\theta$ if $\cos\theta = \frac{\sqrt{3}}{5}$ and $\sin\theta > 0$

$0^\circ < \theta < 90^\circ$

$0^\circ < 2\theta < 180^\circ$



$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cdot\frac{\sqrt{22}}{\sqrt{3}}}{1-\left(\frac{\sqrt{22}}{\sqrt{3}}\right)^2}$$

$$= \frac{2\sqrt{22}}{\sqrt{3}} \div \frac{1-\frac{22}{3}}{1} = \frac{2\sqrt{66}}{3} \div \frac{1-\frac{22}{3}}{1}$$

LCD = 3

$$= \frac{2\sqrt{66}}{3-22} = \frac{2\sqrt{66}}{-19}$$

$$\tan 2\theta = \frac{-2\sqrt{66}}{19}$$

$0^\circ < 2\theta < 180^\circ$

⇒ what quadrant 2θ is located in?

QII

Jan 24-8:27 AM

$\cos 2\theta = \frac{-5}{12} \Rightarrow$ 2θ is in QII

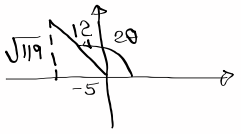
$\sin 2\theta > 0$

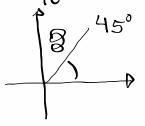
$90° < 2\theta < 180°$

$45° < \theta < 90°$

QI

Find $\sin\theta$





$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\frac{-5}{12} = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 + \frac{5}{12}$

$2\sin^2\theta = \frac{17}{12}$

$\sin^2\theta = \frac{17}{24}$

$\sin\theta = \pm\sqrt{\frac{17}{24}}$

$\sin\theta = \sqrt{\frac{17}{24}}$

$= \frac{\sqrt{17}}{\sqrt{24}} = \frac{\sqrt{17}\sqrt{24}}{24}$

$= \frac{\sqrt{408}}{24}$

$\frac{\sqrt{17}\sqrt{6}\sqrt{4}}{\cancel{24}\ 12}$

$\frac{\sqrt{102}}{12}$

Jan 24-8:36 AM

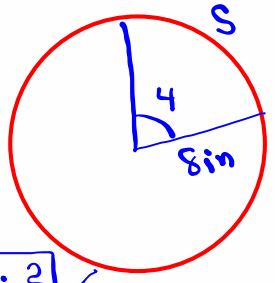
Class QZ 11

The central angle of a circular sector on a circle with radius 8 in has a measure of 4 radians.

1) Draw & clearly label.

2) Find its arc length.

3) Find its area.



$S = r\theta = 8 \cdot 4 = \boxed{32\text{ in}}$ ✓

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(8)^2 \cdot 4 = \boxed{128\text{ in}^2}$ ✓

Jan 24-8:50 AM

Linear Speed

How fast an object moves per time

Speed = $\frac{\text{distance}}{\text{time}}$

$v = \frac{s}{t}$

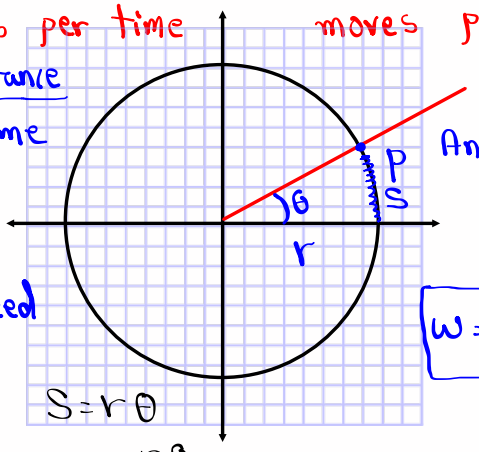
Linear Speed

Angular Speed

How fast an angle moves per time

Angular Speed = $\frac{\text{change } \theta}{\text{time}}$

$\omega = \frac{\theta}{t}$



we know $s = r\theta$

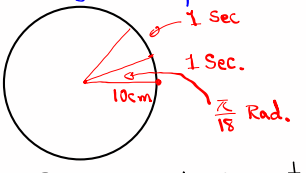
$v = \frac{r\theta}{t} = r\omega$

$v = \frac{s}{t}$ $\omega = \frac{\theta}{t}$ $v = r\omega$

Jan 24-9:16 AM

Suppose P is on a Circle with radius 10cm

The angular speed is $\frac{\pi}{18}$ Radian / Sec.



$r = 10\text{cm}$

$\omega = \frac{\pi}{18}$

Find the angle generated in 6 Seconds.

every Second $\frac{\pi}{18}$

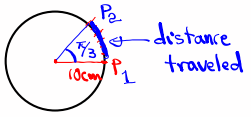
in 6 Seconds $\rightarrow 6 \cdot \frac{\pi}{18} = \frac{\pi}{3}$ Rad.

$\omega = \frac{\theta}{t}$ $\frac{\pi}{18} = \frac{\theta}{6}$ Cross-multiply

$18\theta = 6\pi$

$\theta = \frac{6\pi}{18} = \boxed{\frac{\pi}{3}}$

Find the distance traveled by P in 6 Seconds.



$s = r\theta$

$= 10 \cdot \frac{\pi}{3} = \boxed{\frac{10\pi}{3}\text{ cm}}$

Find its linear speed per Second

$v = \frac{s}{t}$

$= \frac{\frac{10\pi}{3}}{6} = \frac{5\pi}{9}$ cm/Sec.

$\frac{10\pi}{3} \div 6 =$

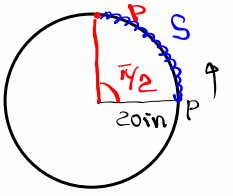
$\frac{5}{\cancel{10}\pi}{3} \cdot \frac{1}{\cancel{6}_{3}}$

Jan 24-9:25 AM

Given $r = 20$ in.

$\omega = \frac{\pi}{12}$ Rad./min.

$t = 6$ min.



1) Find the angle generated by point P in t.

2) Find the distance traveled by P along the circle in time t.

3) Find the linear speed in in/min.

$\frac{\pi}{12} \cdot 6 = \frac{\pi}{2}$ Rad.

$\omega = \frac{\theta}{t}$

$\frac{\pi}{12} = \frac{\theta}{6}$ $\Rightarrow \theta = \frac{\pi}{2}$

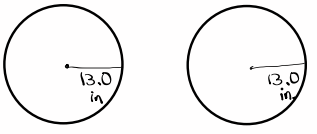
$S = r\theta$

$= 20 \cdot \frac{\pi}{2} = \boxed{10\pi \text{ in}}$

$v = \frac{S}{t} = \frac{10\pi}{6} = \boxed{\frac{5\pi}{3} \text{ in/min.}}$

Jan 24-9:37 AM

The tires on a bike are circular with radius 13.0 in. Tires are turning 215 Revolutions per min.



2π Rad.

$215 \cdot 2\pi$ Rad./min.

$\omega = 430\pi$ Rad./min

$r = 13.0$ in.

How fast is the bike traveling in miles per hour?

Linear Speed

$v = \frac{S}{t}$

$\omega = \frac{\theta}{t}$

1 hr = 60 min.

1 mile = 5280 ft = $5280 \cdot 12$ in.

$v = 13.0 \text{ in.} \cdot \frac{430\pi}{1 \text{ min.}} =$ Miles/hr. $v = r\omega$

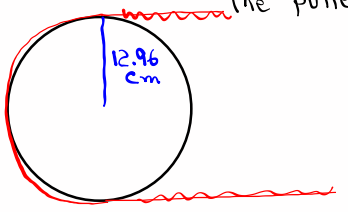
$= 13.0 \text{ in} \cdot \frac{430\pi}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ m}}{12 \cdot 5280 \text{ in}}$ $\boxed{\begin{matrix}1 \text{ hr} = 60 \text{ mins} \\ 1 \text{ Mile} = 5280 \cdot 12 \text{ in}\end{matrix}}$

$= \frac{13.0 \cdot 430\pi \cdot 60}{12 \cdot 5280}$ miles/hr

≈ 16.6 mph

Jan 24-9:46 AM

The pulley shown below has a radius of 12.96 cm. It takes 18 seconds for 56 cm of a belt around the pulley.



Belt is making the pulley to go around.

$t = 18$ Seconds

$S = 56$ cm.

Find linear speed in Cm/Sec.

$v = \frac{S}{t} = \frac{56}{18} = \frac{28}{9}$ cm/sec. ≈ 3.11 cm/sec.

Find its angular speed in radians/sec.

$\omega = \frac{\theta}{t}$ $v = r\omega$ $\frac{28}{9} = 12.96\,\omega$

$\omega = \frac{28/9}{12.96}$

$\approx .24$ Rad./sec.

Jan 24-10:03 AM

Given

$r = 30$ cm

$\omega = \frac{\pi}{10}$ Rad/sec

$t = 4$ Sec.

1) Find angle generated.

$\omega = \frac{\theta}{t}$ $\frac{\pi}{10} = \frac{\theta}{4}$ $\theta = \frac{2\pi}{5}$ Rad.

2) Find distance traveled

$S = r\theta = 30 \cdot \frac{2\pi}{5} = 12\pi$ cm

3) Find linear speed.

$v = \frac{S}{t} = \frac{12\pi}{4} = 3\pi$ cm/Sec

Jan 24-10:13 AM

Find angular speed of an object moving along a circle given $s = \frac{8\pi}{9}$ m, $r = \frac{4}{3}$ m, in $t = 12$ seconds.

$v = \frac{s}{t} \rightarrow v = \frac{\frac{8\pi}{9}}{12} = \frac{8\pi}{12 \cdot 9} = \frac{2\pi}{27}$

$w = \frac{\theta}{t}$

$v = rw$

$\frac{2\pi}{27} = \frac{4}{3} w$

$\frac{2\pi}{27} \cdot \frac{3}{4} = w$

$w = \frac{\pi}{18}$ Rad. Per Sec.

Can you show that $s = rwt$?

$v = \frac{s}{t}$

$w = \frac{\theta}{t}$

$v = rw$

$s = r\theta$ Arc length

$\theta = wt$

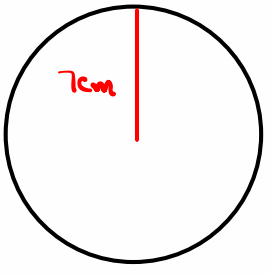
$\frac{8\pi}{9} = \frac{4}{3} \cdot w \cdot 12$

$\frac{8\pi}{9} = 16w$

$w = \frac{\frac{8\pi}{9}}{16} = \frac{8\pi}{9 \cdot 16} = \frac{\pi}{18}$ Rad./Sec

Jan 24-10:18 AM

Consider the tip of the minute hand of a clock if the hand is 7 cm long.



$r = 7$cm

Find the linear speed of the tip of the minute hand in cm per min.

$v = \frac{s}{t}$ $w = \frac{\theta}{t}$ $v = rw$

$v = rw = 7\text{cm} \cdot \frac{\pi}{30}$ Rad/min.

$= \frac{7\pi}{30}$ cm/min.

2π Rad. in 60 min.

$w = \frac{2\pi}{60} = \frac{\pi}{30}$ Rad/min.

Jan 24-10:36 AM

A wheel has a radius of 2 m and it rotates 42 times per minute.

Find its linear Speed.

$r = 2\text{ m}$

$w = 42 \cdot 2\pi$ Rad/min

$= 84\pi$ Rad./min

$v = \frac{s}{t}$, $w = \frac{\theta}{t}$ $v = rw$

$= 2 \cdot 84\pi$

$= \boxed{168\pi \text{ meters/minute}}$

Jan 24-10:44 AM

Introducing D M S

Degree

Minute Portion of degree

Second Portion of Minute

$1^\circ = 60$ Minutes $1^\circ = 60'$

1 Minute $= 60$ Seconds $1' = 60''$

Angle A is $32^\circ\ 15'\ 20''$

$32^\circ\ 15'\ 20'' = 32 + \frac{15}{60} + \frac{20}{3600} \approx \boxed{32.256^\circ}$

Convert $72^\circ\ 8'\ 14''$ to degrees

$72 + \frac{8}{60} + \frac{14}{3600} \approx \boxed{72.137^\circ}$

Convert $\boxed{85^\circ\ 30'}\ 25''$ to degrees

$85 + \frac{30}{60} + \frac{25}{3600} \approx 85.507^\circ$

Jan 24-11:14 AM

Now reverse

Convert 40.125° to DMS

$40^\circ \quad 7' \quad 30''$

$+.125(60) = 7.5'$

$.5(60 = 30$

Convert 34.817° to DMS

$= 34 + .817(60)$

$= 34 + 49.02$

$= 34 + 49 + (.02)(60)$

$\approx 34^\circ \quad 49' \quad 1''$

Convert 50.525° to DMS

$50.525 = 50^\circ + .525(60)'$

$= 50^\circ \quad 31.5' \quad .5(60)$

$= 50^\circ \quad 31' \quad 30''$

Jan 24-11:24 AM

Operations with DMS

$A = 25^\circ \quad 42' \quad 50''$

$B = 32^\circ \quad 17' \quad 10''$

$A+B = 57^\circ \quad 59' \quad 60'' = 58^\circ$

$A = 51^\circ \quad 29'$

$B = 32^\circ \quad 45'$

$A+B = 83^\circ \quad 74'$

$74' = 60' + 14'$ (1°)

$A+B = 84^\circ \quad 14'$

$A = 89^\circ \quad 55' \quad 45''$

$B = 25^\circ \quad 38' \quad 35''$

$A+B = 114^\circ \quad 93' \quad 80''$

$114^\circ \quad 94' \quad 20''$

$115^\circ \quad 34' \quad 20''$

Jan 24-11:32 AM

$A = 35^\circ\ 52'$

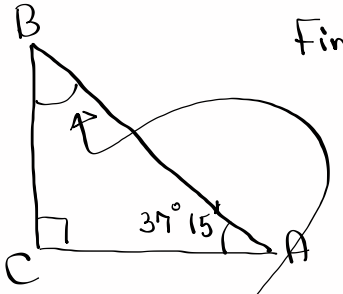
$B = 20^\circ\ 22'$

$A - B = 15^\circ\ 30'$

$A = \cancel{47^\circ}\ 48^\circ\ \cancel{75'}\ 15'$

$B = 32^\circ\ 35'$

$A - B = 15^\circ\ 40'$



Find B

$A + B = 90$

$B = 90^\circ - A$

90°

$-\ 37^\circ\ 15'$

$\Rightarrow$

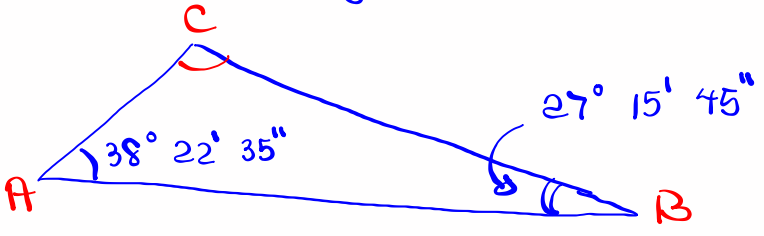
$89^\circ\ 60'$

$-\ 37^\circ\ 15'$

$52^\circ\ 45'$

Jan 24-11:41 AM

Find the missing angle



$A + B = 38^\circ\ 22'\ 35'' + 27^\circ\ 15'\ 45''$

$65^\circ\ 37'\ 80''$

$65^\circ\ 38'\ 20''$

$C = 180^\circ - (A + B)$

$179^\circ\ 59'\ 60''$

$180^\circ\ \cancel{60'}$

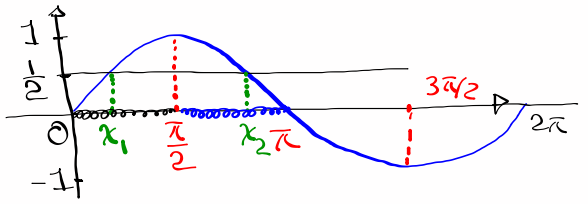
$-\ 65^\circ\ 38'\ 20''$

$C \rightarrow \boxed{114^\circ\ 21'\ 40''}$

Jan 24-11:49 AM

Intro to Trig Equation

Solve $\sin x = \frac{1}{2}$ on $[0, 2\pi)$



Ref. angle → $30° = \frac{\pi}{6}$

QI → $x = \frac{\pi}{6}$

QII → $x = \pi - R.A. = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Solve $\sin x = \frac{1}{2}$ $[0, 2\pi)$

Soln Set $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

Jan 24-11:55 AM

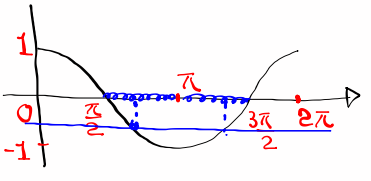
Solve

$2\cos x + \sqrt{3} = 0$

$2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

over $[0, 2\pi)$



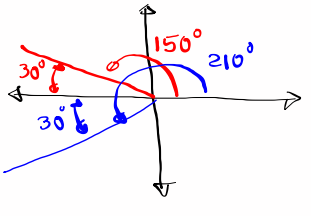
QII , QIII

$\left\{\frac{5\pi}{6}, \frac{7\pi}{6}\right\}$

Ref. angle → $30° = \frac{\pi}{6}$

QII → $x = \pi - R.A. = \pi - \frac{\pi}{6} = \frac{5\pi}{6} = 150°$

QIII → $x = \pi + R.A. = \pi + \frac{\pi}{6} = \frac{7\pi}{6} = 210°$



Jan 24-12:00 PM

Solve

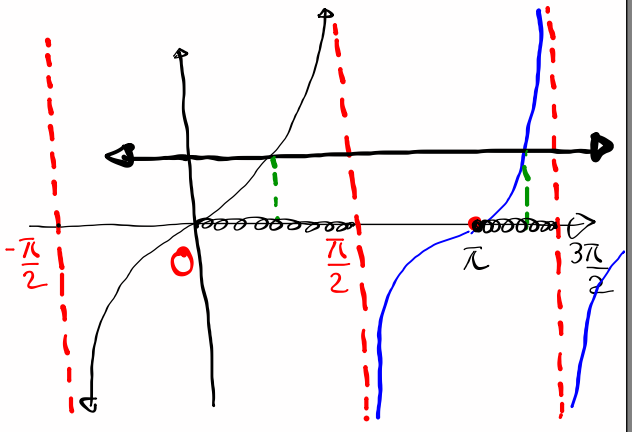
$\tan x - 1 = 0$

over $[0, 2\pi)$

$\tan x = 1$

Ref. angle $\frac{\pi}{4}$

QI , QIII



QI $\quad x = \text{R.A.} = \frac{\pi}{4}$

QIII $\quad x = \pi + \text{R.A.} = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

Jan 24-12:08 PM